BOOTSTRAPPING THE HAUSMAN TEST IN PANEL DATA MODELS

VELIMIR BOLE[1]
PETER REBEC[2]
Economic Institute PF
Prešernova 21
1000 Ljubljana
Slovenia

Key Words and Phrases: Hausman test, Edgeworth expansion, bootstrap.

JEL Classification: C12, C15, C23.

Abstract

The Hausman test statistic in panel data models is asymptotically pivotal under the null hypothesis. It could therefore be refined using the bootstrap resampling technique. Edgeworth expansion shows that coverage of a bootstrap version of the Hausman test is second-order correct. The asymptotic *vis-à-vis* the bootstrap version of Hausman test are compared by Monte Carlo simulations. Results show that the bootstrap version has around 20% lower error in coverage at the null hypothesis. If size-corrected, it also outperforms the power of the asymptotic Hausman test by almost 10% if fixed effects are "weak". Results are robust on parameters of data generating process.

[1]e-mail: velimir.bole@eipf.si, tel.: +386 1 2518704, fax: +386 1 4256870
[2]e-mail: peter.rebec@eipf.si, tel.: +386 1 2521688, fax: +386 1 4256870

1 Introduction

Empirical work with panel data requires a decision on how to treat individual specific effects: whether to use a fixed or random effects model. The decision depends on the correlation between unobserved effect variable and explanatory (independent) variables. The Hausman specification test (Hausman, 1978) is the standard test for discriminating between fixed versus random effects in panel data models. It is asymptotically pivotal under the null hypothesis.

In small samples, the precision of quantiles and coverage, if determined from asymptotic distribution of the Hausman test statistic (HT), could be considerably distorted. Bootstrapping the HT could significantly reduce imprecision, because the statistic is asymptotically pivotal. Edgeworth expansion of the HT could reveal the size of potential corrections (see Hall, 1992, for general discussion on bootstrap and Edgeworth expansion).

The potential advantage of the bootstrap version of the HT would not be limited to the null hypothesis only. The known result of Davidson and MacKinnon (2003), namely, shows that the relative advantage of the bootstrap version of the test (in comparison to the asymptotic version of the test) does not depend on alternatives. The theoretical advantage of the bootstrap version of the test (*vis-à-vis* the asymptotic version), if proved for the null hypothesis, could therefore be directly extended to all alternative hypotheses.

Bootstrapping regression models already has firm theoretical foundation

(Freedman, 1981), while several resampling algorithms for panel data models were suggested by Andersson and Karlsson (2001).

In the present paper, small sample performance of the bootstrap version of the HT is compared to those of the asymptotic version. To document the advantage of bootstrapping the Hausman test statistic and to study the speed of convergence of both versions of the statistic, an Edgeworth expansion of the Hausman test statistic distribution is derived for the analysed model. Empirical results are evaluated by a Monte Carlo experiment on the simple panel data model. Robustness of results is empirically documented for an assumed distribution of individual-specific effect and idiosyncratic error, variance of idiosyncratic error, heteroscedasticity of idiosyncratic error and the size of the correlation between the explanatory (independent) variable and individual-specific effect in the alternative hypothesis.

The structure of the rest of the paper is as follows: In Section 2 we introduce the error-component regression model. In the same section we also derive the Edgeworth expansion for both versions of the HT and give specific theoretical results for the analysed (simplified) model. Section 3 describes the design of the experiment. Details on bootstrap technique and Monte Carlo simulations are given. Specification of a baseline model and alternatives are described as well. In Section 4 experimental results on the size and power of the asymptotic and bootstrap versions of the HT are given for baseline and alternative error-component regression models. In the same section possible theoretical explanation of empirical results is provided. Section 5 concludes the paper. An

Appendix contains the proofs.

2

2.1 Hausman test for panel data

In the error-component regression model

$$y_{it} = \beta_0 + \mathbf{x_{it}}\boldsymbol{\beta} + c_i + u_{it}, \tag{1}$$

$\mathbf{x_{it}}$ is $(1 \times K)$, $\boldsymbol{\beta}$ is $(K \times 1)$, c_i and u_{it} are i.i.d., $cov(c_i, u_{it}) = 0$, $var(c_i) = \sigma_c^2$, $var(u_{it}) = \sigma_u^2$, $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$.

The Hausman specification test for the error-component regression model is based on the (Mahalanobis) distance between fixed ($\hat{\boldsymbol{\beta}}_{\boldsymbol{FE}}$) and random ($\hat{\boldsymbol{\beta}}_{\boldsymbol{RE}}$) effects estimators. Both, $\hat{\boldsymbol{\beta}}_{\boldsymbol{FE}}$ and $\hat{\boldsymbol{\beta}}_{\boldsymbol{RE}}$, estimators are consistent under the null hypothesis

$H_0 : E(c_i|\mathbf{x_i}) = 0.$

For the alternative hypothesis (specified as in Arellano, 1993, p.90)

$H_1 : E(c_i|\mathbf{x_i}) = E(c_i|\bar{\mathbf{x}}_\mathbf{i}) = \bar{\mathbf{x}}_\mathbf{i}^\mathbf{T}\boldsymbol{\gamma},$

only $\hat{\boldsymbol{\beta}}_{\boldsymbol{FE}}$ remains consistent. The test statistic is distributed asymptotically as χ_K^2 under the null hypothesis, where K is the number of unknown parameters. Under the alternative, we have $\boldsymbol{\gamma} \neq 0$ and the test statistic is asymptotically distributed as noncentral χ^2_K.

2.2 Edgeworth expansion of the Hausman test statistic

Edgeworth expansion was originally suggested for simple sample moments, but it also has been derived for more sophisticated statistics, as for example, smooth functions of sample moments (Bhattacharya and Ghosh, 1978), linear regression models (Qumsiyeh, 1990, 1994, 1997 and Hall, 1992) and nonlinear regression models (Ivanov and Zwanzig, 1983 and 2002). None of these generalizations apply directly to the Hausman statistic. However, Edgeworth expansion for regression models can be applied to the HT with some modifications.

In the following two propositions, the basic characteristics of Edgeworth expansion of the (bootstrapped) Hausman test statistic distribution necessary for the discussion of empirical results are given. In the rest of the paper, Edgeworth expansion of only the t-percentile version of the HT are discussed, without special notice.

Proposition 1 *Assume that in model (1), Cramér's condition*

$$\limsup_{\xi\to\infty} |E(exp(j\xi c_i))| < 1, \;\; \limsup_{\xi\to\infty} |E(exp(j\xi u_{it}))| < 1, \quad (j = \sqrt{-1}),$$

and

$$E(|c_i|^l) < \infty, E(|u_{it}|^l) < \infty \; for \; l \leq 10$$

holds for errors c_i and u_{it}, $i = 1, ..., N$, $t = 1, ..., T$. Suppose also that vectors of the predetermined variables $\{x_{it} : i = 1, ..., N\ ,\ t = 1, ..., T\}$ represent the sequence of independent realizations of a random vector X with $E(||X||^m) < \infty$, $m \leq 5$, $|cov(X)| > 0$. Then, the distribution of the HT under the null hypothesis permits the Edgeworth expansion

$$\int_E \left(1 + \sum_{i=1}^{2} N^{-i/2} Q_i(w)\right) \phi(w) dw \tag{2}$$

with error $O(N^{-3/2})$ when $N \to \infty$; Q_i denote polynomials of degree $3i$ with the same parity as i; if E is sphere in $\mathcal{R}^K$ only Q_2 matters. If moment conditions are strengthened to high enough l, the bootstrap version of the HT under the null hypothesis permits the Edgeworth expansion

$$\int_E \left(1 + \sum_{i=1}^{2} N^{-i/2} \hat{Q}_i(w)\right) \phi(w) dw$$

with error $O(N^{-3/2})$ and second-order correct coverage when $N \to \infty$; $\hat{Q}_i$ denote polynomials as in (2), only distribution parameters, if they figure in polynomials Q_i, are replaced by consistent estimates.

In the Appendix, it is shown that existing theory on Edgeworth expansion of regression models (see Hall, 1992) can be applied to prove the proposition. Let us add that stated Cramér's condition holds if the distribution of c_i (u_{it}) is nonsingular (for example, if it possesses proper density function).

In the case of the baseline model studied in the experiment, an even stronger result on the coverage of the bootstrap version of the HT could be proved.

Proposition 2 *In the error-component regression model (1) with normally distributed c_i and u_{it}, $K = 1$, and $n = NT$, the bootstrap version of the Hausman test statistics is second-order correct; that is, the size of the bootstrap version of the HT is correct including Edgeworth terms of order n^{-1}. If ξ_α^2 is α quantile of the χ^2 -distribution for nominal size α, then the effective size of the test equals*

$$\alpha_{eff} = \alpha + \frac{2}{n}\left(\xi_\alpha\left(2 + \frac{1}{4}(\xi_\alpha^2 - 3)\right)\phi(\xi_\alpha)\right) + O(n^{-3/2}), \tag{3}$$

where $\phi(\xi_\alpha)$ is the value of the density function for the standardized normal variable in ξ_α.

The proposition is proved in the Appendix.

Because of the Davidson and MacKinnon result (2003), the precision of the coverage of the bootstrapped statistic under the null hypothesis can be extended to all alternatives! In the analysed baseline and alternative models, therefore, the coverage of the bootstrap version of the HT is second-order correct.

3 Design of the simulation experiment

3.1 Model variants

Three groups of model variants are analysed.

In the main group, baseline and several alternative models are compared. Each alternative differs from the baseline model only in one parameter of the data generating process. Baseline and alternative models are simulated with and without fixed effects.

In all variants from the main group, the panel has 25 cross-section units and 10 time observations. Correlation between the time-averaged explanatory variable (for every cross-section unit) and the individual specific effect equals 0.5 when simulated with fixed effects and, obviously, 0.0 if simulated without fixed effects.

Specification of model characteristics for the model variants from the main group is presented in Table 1. Every line gives parameters for one variant. The main characteristic (difference *vis-à-vis* the baseline model) is given in the first column as an indicator of the corresponding variant. In other columns, analysed characteristics are given - one characteristic per column.

In the second and third columns, distributions of individual-specific effect and idiosyncratic error are given, respectively. The number of explanatory variables is presented in the fourth column and denoted by K. The generating process for explanatory variable(s) is described in the fifth and sixth column. The cross-section heteroscedasticity of u_{it} is defined so that half of the cross-section units have variance equal to 0.5 and the other half equal to 1. In Table 1, the distribution of u_{it} is therefore denoted by $N(0, 0.75)$, when heteroscedasticity is present.

To study more thoroughly the effects of "fixed effects intensity" and magnitude of nominal size on the relative advantage of bootstrap *vis-à-vis* asymptotic versions of the HT, two additional groups of model variants are analysed. In the first group, the correlation between the time-averaged explanatory variable and individual specific effect ("fixed effects intensity") takes the values $\pm 0.7, \pm 0.5, \pm 0.3$ and 0; all other parameters are equal, as in the baseline model. In the second group of alternative models, the analysed magnitude of nominal size takes the values $0.05, 0.10, 0.15$ and 0.20; other parameters are again equal, as in the baseline model.

3.2 Bootstrap procedure

A model-based non-parametric bootstrap was applied to the one-way error component regression model with individual specific effects, as proposed by Andersson and Karlsson (2001). Bootstrap samples are drawn with replacements from N individual specific components (c_i), as well as from NT idiosyncratic error terms (u_{it}).

Because the vector of parameters $\boldsymbol{\beta}$ is not known, the consistent estimate $\hat{\boldsymbol{\beta}}$ is used to obtain both residual components according to

$\hat{u}_{it} = (y_{it} - \bar{y}_i) - (\mathbf{x}_{it} - \bar{\mathbf{x}}_i)\hat{\boldsymbol{\beta}}$, and $\hat{c}_i = \bar{y}_i - \bar{\mathbf{x}}_{\mathbf{i}}\hat{\boldsymbol{\beta}}$.

Once the bootstrap samples of residual components are obtained, values for y^*_{it} are generated by model (1). Then parameters of the model and bootstrap values of the Hausman statistic are calculated. Repeating the generation of bootstrap samples and computing the HT value in each bootstrap repetition gives the distribution of the bootstrapped HT statistic.

Heuristically speaking, non-parametric bootstrap resampling of the error components corresponds to the basic idea of the Hausman test. The very resampling of errors, namely, "disconnects" the individual-specific effect component and explanatory variables.

3.3 Monte Carlo simulations

Monte Carlo simulation is performed to evaluate the size and power of asymptotic and bootstrap versions of the HT for all model variants. The size of both versions of the HT is analysed on the model generated without fixed effects, and power on the (otherwise equal) model generated with fixed effects.

Monte Carlo simulation of the particular model variant is made in one shot. That is, in every run, asymptotic and bootstrap versions of the HT are calculated. First, all runs for the model without fixed effects are completed; then, runs for the (otherwise equal) model with fixed effects are made.

In order to achieve comparability of power between two versions of the HT, the size-corrected power of the bootstrapped test is used (see, Davidson and MacKinnon, 2003). Therefore, at every run of a particular Monte Carlo simulation, when analysing the model with fixed coefficients, the nominal size of the bootstrapped version of the HT is corrected to the effective size of the asymptotic version of the HT, calculated in the same Monte Carlo simulation, after all runs for the (otherwise equal) model without fixed effects are made.

3.4 Monte Carlo parameters

The number of runs in a particular Monte Carlo simulation is set to 10,000. This number was determined on the basis of experimental evidence. Figure 1 shows the results for size of two simulations of the model with the same data-generating process. Heuristically, convergence of both experiments is attained at 10,000 runs. The same number of runs was also used in the paper of Andersson and Karlsson (2001).

Davidson and MacKinnon (2000) show that, with bootstrapping, 399 replications are about the minimum for a test at the 0.05 level in order to avoid a power loss of more than 1%. The number of bootstrap replications should be chosen so that $\alpha(B+1)$ is an integer, if the test is to be exact. However, in our study, the difference between 399 and 400 replications was within the convergence deviation illustrated in Figure 1[3]. Empirical distribution of the Hausman statistic is generated by 400 bootstrap replications.

An average Monte Carlo simulation (resulting in an empirical distribution of the asympthotic and bootstrap versions of the HT, for models with and without fixed coefficients) took about 170 hours on a 1Ghz PC with 256Mb of RAM in the Matlab environment.

[3] Corresponding results are available from the authors upon request.

4 Discussion of empirical reults

4.1 Results

The size and power of asymptotic and bootstrap versions of the HT for baseline and alternative models are given in Table 2. Model variants are indicated in the first column.

The size of asymptotic and bootstrap versions of the HT exceed nominal size in all model variants. Relative error in the coverage-(effective)size of the asymptotic test is approximately 20% larger than for the boostrap test. The size-corrected power of the bootstrap test is almost the same as the power of the asymptotic test.

Sensitivity of size and power on changes of DGP parameters is small. Only increasing the variance of idiosyncratic error has a slight effect on deterioration of power. Changes in analysed parameters do not cause any visible differential effect in size or power of the asymptotic *vis-à-vis* the bootstrap version of the HT.

Table 3 shows the effects of nominal size magnitude on coverage (size and power) of both versions of the HT. For all cases, the size of the asymptotic version exceeds that of the bootstrap version of the HT. The asymptotic test also systematically overshoots nominal size, while the bootstrap version of the HT does not. Reduction of the error in coverage for the bootstrapped HT is largest for the nominal size 0.1. Differences in power between asymptotic and size-corrected bootstrap versions of the HT are negligible for all analysed nominal sizes.

The power function of the bootstraped version of the HT is shown in Figure 2. Its position and shape indicate good power characteristics.

Table 4 shows the influence of correlation between the time-averaged explanatory variable and individual-specific effect on power of the HT. This, therefore, illustrates the importance of the "intensity of fixed effects" for the relative power of bootstrap *vis-à-vis* asymptotic versions of the HT. Relative power of the bootstrap (in comparison to the asymptotic) version of the HT is also illustrated (with a fitted curve) for the extended sample of variants in Figure 3[4].

The power of asymptotic and bootstrapped versions of the HT is, as heuristically expected, symmetric and increases approximately with the square of the correlation. The relative advantage of the bootstrap version of the HT is greater for smaller absolute values of the correlation, but the increase is not monotone, as Figure 3 demonstrates. The bootstrap version of the HT outperforms the asymptotic version to the greatest extent around an absolute correlation of 0.1, where the size-corrected power of the bootstrap version is almost 10% higher than the power of the asymptotic version of the HT.

[4]Because of the time-consuming computation, additional Monte Carlo simulations (except those already presented in Table 4) are calculated with only 5000 runs.

4.2 Discussion

The bootstrap version of the HT outperforms the asymptotic version for all model variants studied. Coverage error is reduced from around 0.012, for asymptotic HT, to around 0.003 for the bootstrap verison, for null hypothesis and nominal size 0.05. The correction term from the Edgeworth expansion (0.004) accounts for approximatelly half of the error reduction.

Power differences between asymptotic and size-corrected bootstrap versions of the HT are negligible. Such empirical findings are in line with the theoretical results of Davidson and MacKinnon (2003).

The advantage of the bootstrap version of the HT is robust on studied DGP parameters. Such empirical results can be, heuristically speaking, expected, because Edgeworth correction (including the second-order term for bootstrapped HT distribution in the analysed model) does not depend on any DGP parameter.

In the empirical experiment, reduction of error in coverage in the bootstrapped HT (in comparison with the asymptotic version of the HT) depends on the magnitude of nominal size. The highest value is attained at 0.1. Again, Edgeworth expansion terms can be used for heuristic interpretation of empirical results. But, illustration is not precise enough. The correction term in Edgeworth expansion (2) confirms, that reduction of error in coverage does depend on nominal size. The maximum of the correction term in Edgeworth expansion (2) is, namely, attained at $\xi_\alpha^2 = 1.45$, which corresponds to the nominal size 0.2. Probably, third term in Edgeworth expansion would have to be taken into account also.

Differences in the power of size-corrected bootstrapped HT and the asymptotic version of the HT are negligible in the main group of analysed model variants (including baseline model). For these alternatives, however, correlation of the time-averaged explanatory variable and individual-specific effect is high (equal to 0.5). Analysing the "intensity of fixed effects" more in detail shows the potential additional advantage of the bootstrap version of HT *vis-à-vis* the asymptotic version of the HT. The empirical result shows that in the panel data model with a weak "intensity of fixed effects", bootstrapped HT significantly outperforms asymptotic HT, also on testing the alternative hypothesis and after size correction. Namely, the power of the bootstrap test is almost 10% higher than the power of the asymptotic test. Such relative advantage of the bootstrapped HT "near" the null hypothesis could be explained by Davidson and MacKinnon result on drifting hypothesis (2003, Theorem 1) if position of peaks on Figure 3 migrate (with $n^{1/2}$) to 0. Empirical verification of such possible explanation can be an incentive for additional future research.

5 Conclusion

Using an asymptotic distribution of the Hausman specification test in the error-component regression model entails considerable size and power distortions in small samples. Because of pivotalness, bootstrapping the

Hausman test would have to mitigate those distortions. Edgeworth expansion shows that coverage of the bootstrap version of Hausman test is second-order correct under the null hypothesis. Empirical results confirm theoretically expected results. The bootstrap version of the Hausman statistic has systematically lower error in coverage at the null hypothesis. The power of the size-corrected bootstrap test and asymptotic tests differ only negligibly for "normal" intensity of fixed effects. Such an empirical finding is in line with the theoretical results of Davidson and MacKinnon. Advantages of bootstrapping the Hausman test are greatest on the margin, when the "intensity of fixed effects" is weak (correlation of individual effects and time-averaged explanatory variables is not high). Its power is almost 10% higher than in the case of the asymptotic test. Relative performance of the bootstrap version of the Hausman test are robust on distribution parameters of both components of error.

A Appendix

Proof of Proposition 1 *The error-component regression model can be written in the form*

$$\mathbf{y} = \beta_0 + \mathbf{X}\boldsymbol{\beta} + \mathbf{v} \tag{4}$$

where the vector of errors is distributed $\mathbf{v} \sim N(0, \boldsymbol{\Sigma})$, $\boldsymbol{\Sigma} = \sigma_u^2(\frac{1}{\psi^2}\mathbf{P} + \mathbf{Q})$, *the parameter* ψ *is defined by* $\psi = \frac{\sigma_u}{(T\sigma_c^2 + \sigma_u^2)^{1/2}}$, $\mathbf{P}$ *and* $\mathbf{Q}$ *are between and within projectors* $(\mathbf{P} = \mathbf{I}_N \otimes \bar{\mathbf{J}}_T, \mathbf{Q} = \mathbf{I} - \mathbf{P})$, $\mathbf{y}$ *is* $(NT \times 1)$ *vector,* $\mathbf{X}$ *is* $(NT \times K)$ *matrix of independent variables, and* $\boldsymbol{\beta}$ *is* $(K \times 1)$ *vector.*

Testing the null hypothesis (of random effects) with the HT in such model is equivalent to the Wald test of the hypothesis that parameter $\boldsymbol{\theta}$ is zero in the model

$$\mathbf{y} = \beta_0 + \mathbf{X}\boldsymbol{\beta} + \mathbf{Q}\mathbf{X}\boldsymbol{\theta} + \mathbf{v} \tag{5}$$

using the Aitkin estimator for $\boldsymbol{\theta}$ (see equivalent transformation in Baltagi, 2001, page 66). Because sectoral errors $\mathbf{v_i} = [c_i + v_{i1}, c_i + v_{i2}, ..., c_i + v_{iT}]^T$ are i.i.d. vectors, the error-component regression model (5) can be written in the form

$$\mathbf{y_i} = \beta_0 + \mathbf{X_i}\boldsymbol{\beta} + (\mathbf{I_T} - \bar{\mathbf{J}}_\mathbf{T})\mathbf{X_i}\boldsymbol{\theta} + \mathbf{v_i}, \;\; cov(\mathbf{v}_i) = \sigma_v^2 \left(\frac{1}{\psi^2}\mathbf{J_T} + \left(\mathbf{I_T} - \bar{\mathbf{J}}_\mathbf{T}\right) \right) \tag{6}$$

where $\mathbf{y_i} = [\mathbf{y_{i1}}, \mathbf{y_{i2}}, ..., \mathbf{y_{iT}}]^\mathbf{T}$ and $\mathbf{X_i} = [\mathbf{x_{i1}^T}, \mathbf{x_{i2}^T}, ..., \mathbf{x_{iT}^T}]^\mathbf{T}$ are cross-section blocks of the dependent variable vector and explanatory variables matrix.

To multivariate multiparameter regression model (6) can be applied conclusions of Hall (1992), section 4.3.6, on t-percentile testing (confidence) regions for slope. Because the testing region for the Hausman test statistics is symmetric and parameters of the second $\left(N^{-1}\right)$ term in the Edgeworth expansion of the bootstrapped HT are correct with error of order $O\left(N^{-1/2}\right)$, the conclusion on the coverage follows. The application of theorem 5.4 from Hall (1992) in the vector case also gives the existence of the Edgeworth expansion.

Proof of Proposition 2 *Because errors are normally distributed, the Hausman test is equivalent to the Wald test*

$$\mathbf{y}^* = \boldsymbol{\beta}_\mathbf{0}^* + \mathbf{X}^*\boldsymbol{\beta} + \tilde{\mathbf{X}}\boldsymbol{\gamma} + \mathbf{w}, \tag{7}$$

where $\mathbf{y}^* = \sigma_u \mathbf{\Sigma}^{-1/2}\mathbf{y}$ *is* $(n \times 1)$ *vector,* $\mathbf{X}^* = \sigma_u \mathbf{\Sigma}^{-1/2}\mathbf{X}$ *is* $(n \times K)$ *matrix,* $\tilde{\mathbf{X}} = \mathbf{QX}$ *is* $(n \times K)$ *matrix, and* $\mathbf{w} \sim N(0, \sigma_u^2)$ *is* $(n \times 1)$ *vector, and, of course,* $n = NT$ *(Baltagi, 2002, p.69). Because of Frisch-Waugh-Lovell theorem testing the hypothesis on* $\boldsymbol{\gamma}$ *in model (7) is equivalent to testing in the model*

$$\mathbf{UM_{X^*}y^*} = \mathbf{UM_{X^*}\tilde{X}}\boldsymbol{\gamma} + \mathbf{v2} \tag{8}$$

where $\mathbf{M_{X^*}}$ *is projector on* $[\mathbf{e}\ \mathbf{X}^*]^\perp$ *and* $\mathbf{U}$ *orthogonal matrix diagonalising* $\mathbf{M_{X^*}}$*. Errors* $\mathbf{v2}$ *are i.i.d. and distributed according to* $N\ (0, \sigma_u^2)$*. Because* $K = 1$*, the conclusions of Hall (1989) or Hall (1992) on the Edgeworth expansion of the t-percentile statistic for the regression slope can be used in the model (8).*

Because the testing region (equivalent to the HT) for the hypothesis $\boldsymbol{\gamma} = 0$ *is symmetric, odd terms in the Edgeworth expansion vanish. For coverage of the HT, therefore, only the second term (out of the first three terms) matters in the Edgeworth expansion, that is*

$q_2(\xi) = -\xi\left(2 + \frac{1}{24}(\kappa\kappa_Z + 6)(\xi^2 - 3) + \frac{1}{72}\lambda\lambda_Z^2(\xi^4 - 10\xi^2 + 15)\right)$

and

$\hat{q}_2(\xi) = -\xi\left(2 + \frac{1}{24}(\hat{\kappa}\kappa_Z + 6)(\xi^2 - 3) + \frac{1}{72}\hat{\lambda}\lambda_Z^2(\xi^4 - 10\xi^2 + 15)\right)$

in the bootstrap version (see, Hall, 1992), where λ, κ *are asymmetric and skewness indicators of standardized error,* λ_Z, κ_Z *sample asymmetry and skewness indicators for explanatory variable* Z *and* $\hat{\lambda}, \hat{\kappa}$ *sample version of error moments. Because errors are normally distributed and sample moments* *converge* ($n^{-1/2}$) *to population moments, for the known* ψ *only*

$$q_2(w) = -w(2 + \tfrac{1}{4}(w^2 - 3))$$

matters in both version of Edgeworth expansion. Corresponding Edgeworth correction is third order correct for t-percentile HT and second order correct for the bootstrap version of HT.

Because the parameters of $q_2(w)$ *are known (do not depend on any parameter of the model), the conclusion of Propostion 2 follows.*

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Table 1

Specification of the main group model variants

	c_i	u_{it}	K	$x1_{it}$	$x2_{it} + 0.8x2_{it-1}$
baseline model	$N(0, 0.5)$	$N(0, 0.5)$	1	$N(0, 1)$	-
c_i $U(0, 0.5)$	$U(0, 0.5)$	$N(0, 0.5)$	1	$N(0, 1)$	-
u_{it} $U(0, 0.5)$	$N(0, 0.5)$	$U(0, 0.5)$	1	$N(0, 1)$	-
$var(u_{it}) = 1$	$N(0, 0.5)$	$N(0, 1)$	1	$N(0, 1)$	-
cross-section heterosced. u_{it}	$N(0, 0.5)$	$N(0, 0.75)$	1	$N(0, 1)$	-
2 explanat. var.	$N(0, 0.5)$	$N(0, 0.5)$	2	$N(0, 1)$	$N(0, 0.36)$

Table 2

Size and power of the Hausman test

	size of test[a]		*power of test*	
	asymptotic	*bootstrap*	*asymptotic*	*bootstrap*[b]
baseline model	0.0622	0.0519	0.7771	0.7782
c_i $U(0, 0.5)$	0.0637	0.0547	0.7653	0.7709
u_{it} $U(0, 0.5)$	0.0620	0.0520	0.7680	0.7719
$var(u_{it}) = 1$	0.0612	0.0534	0.7218	0.7205
cross-section				
heterosced. u_{it}	0.0606	0.0518	0.7458	0.7453
2 explanat. var.	0.0682	0.0527	0.5104	0.5087

[a]Nominal size: α=0.05.
[b]Power of size-corrected bootstrap test.

Table 3

Effects of nominal size magnitude on size and power of the Hausman test

	size of test		*power of test*	
	asymptotic	*bootstrap*	*asymptotic*	*bootstrap*[a]
$\alpha = 0.05$	0.0622	0.0519	0.7771	0.7782
$\alpha = 0.10$	0.1102	0.0967	0.8494	0.8469
$\alpha = 0.15$	0.1647	0.1550	0.8960	0.8964
$\alpha = 0.20$	0.2083	0.1994	0.9216	0.9212

[a]Power of size-corrected bootstrap test.

Table 4

Effects of correlation ("fixed effects intensity")

on power of the Hausman test

	power of test	
	asymptotic	*bootstrap*[a]
r = -0.7	0.9967	0.9969
r = -0.5	0.7621	0.7655
r = -0.3	0.3392	0.3455
r = 0	0.0622	0.0519
r = 0.3	0.3361	0.3404
r = 0.5	0.7771	0.7782
r = 0.7	0.9951	0.9953

[a]Power of size-corrected bootstrap test.
Remark: r = correlation between time-averaged explanatory variable and individual-specific effect.

Figure 1

Coverage-size convergence for bootstrap and asymptotic versions of the HT

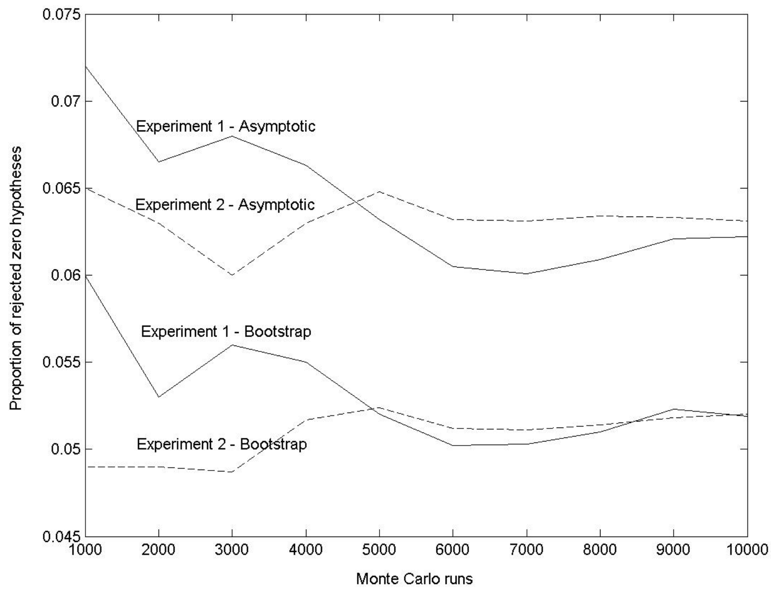


Figure 2

Power as a function of size for the bootstrap version of the HT

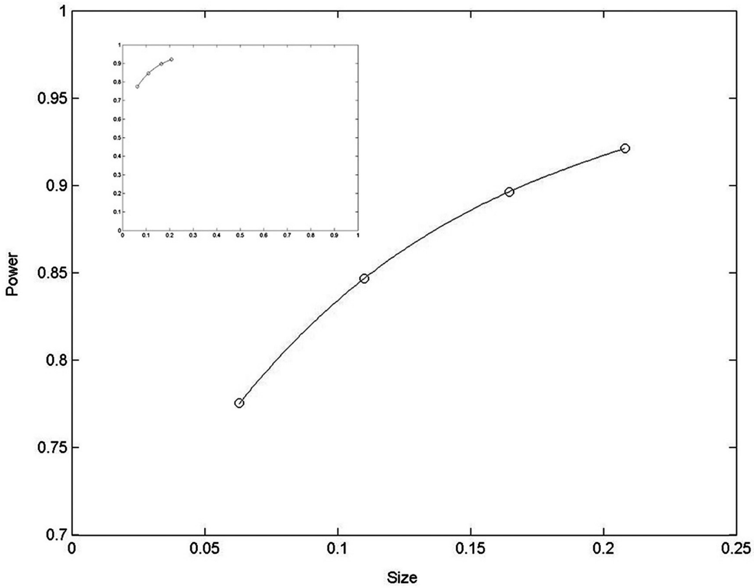
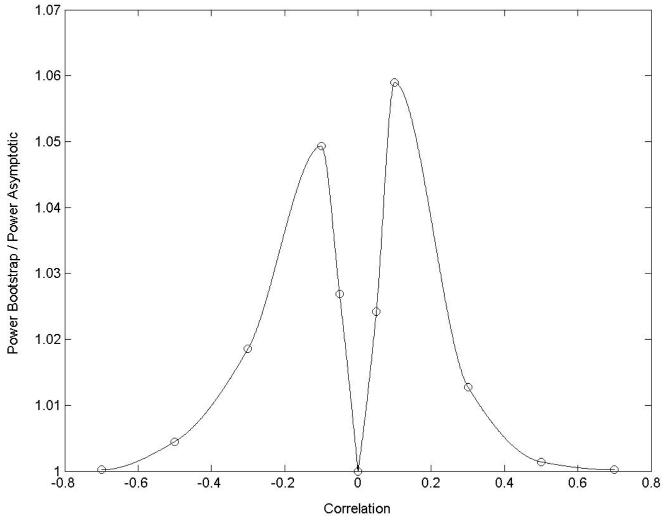


Figure 3
RELATIVE POWER ADVANTAGE OF BOOTSTRAP HAUSMAN TEST[a]



[a]Size-corrected bootstrap test.
Remark: Correlation between time averaged explanatory variable and individual-specific effect.